THE CARMICHAEL NUMBERS UP TO 10^{18}

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ABSTRACT. We extend our previous computations to show that there are 1401644 Carmichael numbers up to 10^{18}. As before, the numbers were generated by a back-tracking search for possible prime factorisations together with a "large prime variation". We present further statistics on the distribution of Carmichael numbers.

1. INTRODUCTION

A *Carmichael number* N is a composite number N with the property that for every b prime to N we have $b^{N-1} \equiv 1 \mod N$. It follows that a Carmichael number N must be square-free, with at least three prime factors, and that $p-1|N-1$ for every prime p dividing N: conversely, any such N must be a Carmichael number.

For background on Carmichael numbers and details of previous computations we refer to our previous paper [4]: in that paper we described the computation of the Carmichael numbers up to 10^{15} and presented some statistics. These computations have since been extended to 10^{16} [5], 10^{17} [6] and now to 10^{18}, using similar techniques, and we present further statistics.

The complete list of Carmichael numbers up to 10^{18} is available from the author's web site at `www.chalcedon.demon.co.uk`.

2. ORGANISATION OF THE SEARCH

We used improved versions of strategies first described in [4].

The principal search was a depth-first back-tracking search over possible sequences of primes factors $p_1, \ldots, p_d$. Put $P_r = \prod_{i=1}^{r} p_i$, $Q_r = \prod_{i=r+1}^{d} p_i$ and $L_r = \operatorname{lcm}\{p_i - 1 : i = 1, \ldots, r\}$. We find that Q_r must satisfy the congruence $N = P_r Q_r \equiv 1 \bmod L_r$ and so in particular $Q_d = p_d$ must satisfy a congruence modulo L_{d-1}: further $p_d - 1$ must be a factor of $P_{d-1} - 1$. We modified this to terminate the search early at some level r if the modulus L_r is large enough to limit the possible values of Q_r, which may then be factorised directly.

We also employed the variant based on proposition 2 of [4] which determines the finitely many possible pairs (p_{d-1}, p_d) from P_{d-2}. In practice this was useful only when $d = 3$ allowing us to determine the complete list of Carmichael numbers with three prime factors up to 10^{18}. The count of these is 35586 (not 35585 as incorrectly reported in our preliminary work).

Finally we employed a different search over large values of p_d, in the range $10^7 < p_d < 10^9$, using the property that $P_{d-1} \equiv 1 \bmod (p_d - 1)$ and factorising the numbers in this arithmetic progression less than $10^{18}/p_d$.

3. STATISTICS

We have shown that there are 1401644 Carmichael numbers up to 10^{18}, all with at most 11 prime factors. We let $C(X)$ denote the number of Carmichael numbers less than X and $C(d, X)$ denote the number with exactly d prime factors. Table 1 gives the values of $C(X)$ and Table 2 the values of $C(d, X)$ for X in powers of 10 up to 10^{18}.

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n	$C(10^n)$
3	1
4	7
5	16
6	43
7	105
8	255
9	646
10	1547
11	3605
12	8241
13	19279
14	44706
15	105212
16	246683
17	585355
18	1401644

TABLE 1. Distribution of Carmichael numbers up to 10^{18}.

X	3	4	5	6	7	8	9	10	11	total
3	1	0	0	0	0	0	0	0	0	1
4	7	0	0	0	0	0	0	0	0	7
5	12	4	0	0	0	0	0	0	0	16
6	23	19	1	0	0	0	0	0	0	43
7	47	55	3	0	0	0	0	0	0	105
8	84	144	27	0	0	0	0	0	0	255
9	172	314	146	14	0	0	0	0	0	646
10	335	619	492	99	2	0	0	0	0	1547
11	590	1179	1336	459	41	0	0	0	0	3605
12	1000	2102	3156	1714	262	7	0	0	0	8241
13	1858	3639	7082	5270	1340	89	1	0	0	19279
14	3284	6042	14938	14401	5359	655	27	0	0	44706
15	6083	9938	29282	36907	19210	3622	170	0	0	105212
16	10816	16202	55012	86696	60150	16348	1436	23	0	246683
17	19539	25758	100707	194306	172234	63635	8835	340	1	585355
18	35586	40685	178063	414660	460553	223997	44993	3058	49	1401644

TABLE 2. Values of $C(X)$ and $C(d,X)$ for $d \le 10$ and X in powers of 10 up to 10^{18}.

We have used the same methods to calculate the smallest Carmichael number S_d with d prime factors for d up to 35. The results are given in Table 3. Heuristics suggest that a good approximation for $\log S_d$ might be $2\log 2(d-1)\log(d-1)$. There is some support for this from Table 4.

In Table 5 and Figure 1 we tabulate the function $k(X)$, defined by Pomerance, Selfridge and Wagstaff [7] by

$$C(X) = X \exp\left(-k(X)\frac{\log X \log\log\log X}{\log\log X}\right).$$

They proved that $\liminf k \ge 1$ and suggested that $\limsup k$ might be 2, although they also observed that within the range of their tables $k(X)$ is decreasing: Pomerance [8],[9] gave a heuristic argument suggesting that $\lim k = 1$. The decrease in k

d	N
3	561
4	41041
5	825265
6	321197185
7	5394826801
8	232250619601
9	9746347772161
10	1436697831295441
11	60977817398996785
12	7156857700403137441
13	179156281066258576752 1
14	87674969936234821377601
15	65531309267520060314817 61
16	15902312310431783769516984 01
17	35237869211718889547310642241
18	32809426840359564991177172754241
19	28108645626353684260052681426160 01
20	34940751534228743505060320471958720 1
21	12586188784963996984763868103868078736 1
22	12758106140074522771498516740500829830401
23	23333793365462164081311115337105403499032 01
24	29457179106737538988590723908950340861856000 1
25	13091296197431676772386520145418795505617841560 1
26	13513093081489380840188651246675032067011140079201
27	74828959377132623928833069491729170489280681292064 01
28	13203403544774501706822913298301389472256950295362816 01
29	37938238144739952732261846613015466851265291071422420960 1
30	70416887142533176417390411931483993124120785701395296424001
31	28841675095935814802054746276846860086244831478146478414368 01
32	47548683776010467321199338399813630819720149485225108264177840 01
33	13347338771470623824869348071051978994960022011138499204965105416 01
34	26084932307537183566978409438381212035926078381015722573062338838240 1
35	11250538045029660697033845962998878260425203320935001088822714733812000 1

TABLE 3. The smallest Carmichael numbers with d prime factors for d up to 35.

is reversed between 10^{13} and 10^{14}: see Figure 1. We find no clear support from our computations for any conjecture on a limiting value of k.

In Table 5 we also give the ratios $C(10^n)/C\big(10^{n-1}\big)$ investigated by Swift [11]. Swift's ratio, again initially decreasing, also increases again before 10^{15}.

In Table 6 and Figure 2 we see that within the range of our computations $C(X)$ is a slowly growing power of X: about $X^{0.34}$ for X around 10^{18}.

In Table 7 we give the number of Carmichael numbers in each class modulo m for $m = 5$, 7, 11 and 12.

In Tables 8 and 9 we give the number of Carmichael numbers divisible by primes p up to 97. In Table 8 we count all Carmichael numbers divisible by p: in Table 9 we count only those for which p is the smallest prime factor.

The largest prime factor of a Carmichael number up to 10^{18} is $p = 704988733$, dividing $n = 994018226608901845 = 5 \cdot 13 \cdot 1733 \cdot 12517 \cdot 704988733$. We have $p = n^{0.4916}$: indeed $n = p(2p-1)$ and $n - 1 = (p-1)(2p+1)$.

d	$\frac{\log(S_d)}{2\log 2(d-1)\log(d-1)}$
3	3.293621187
4	2.324869052
5	1.772215418
6	1.755823184
7	1.503593258
8	1.385943139
9	1.296862582
10	1.273113126
11	1.210794211
12	1.187291825
13	1.183842403
14	1.142841324
15	1.115637251
16	1.112255293
17	1.068846695
18	1.086833722
19	1.067861876
20	1.055266652
21	1.056211783
22	1.041906608
23	1.034833298
24	1.024202006
25	1.026042173
26	1.014073506
27	1.017122489
28	1.010169037
29	1.007224804
30	1.000945160
31	0.984182030
32	0.993535534
33	0.990337138
34	0.984854273
35	0.984296312

TABLE 4. The smallest Carmichael number with d prime factors S_d compared to $2\log 2(d-1)\log(d-1)$.

The largest prime to occur as the smallest prime factor of a Carmichael number in this range is 891001, dividing

$$804230935331967001 = 891001 \cdot 895051 \cdot 1008451.$$

We note that this number is of the form $(220+1)(221+1)(249+1)$ with $k = 4050$.

We define the *index* of a Carmichael number N to be the integer $i(N) = (N-1)/\lambda(N)$ where λ is the Carmichael function, the exponent of the multiplicative group. A result of Somer [10] implies that $i(N) \to \infty$ as N runs through Carmichael numbers. In Table 10 we list the Carmichael numbers known to have index less than 100: the list is complete for numbers up to 10^{18}.

We define the *Lehmer index* of a number N to be $\ell(N) = (N-1)/\phi(N)$. Lehmer's totient problem asks whether there is a number of integral Lehmer index greater than 1 (that is, whether there is a composite N such that $\phi(N)$ divides $N-1$). Such a number would have to be a Carmichael number. See Guy [2] §B37 for further details.

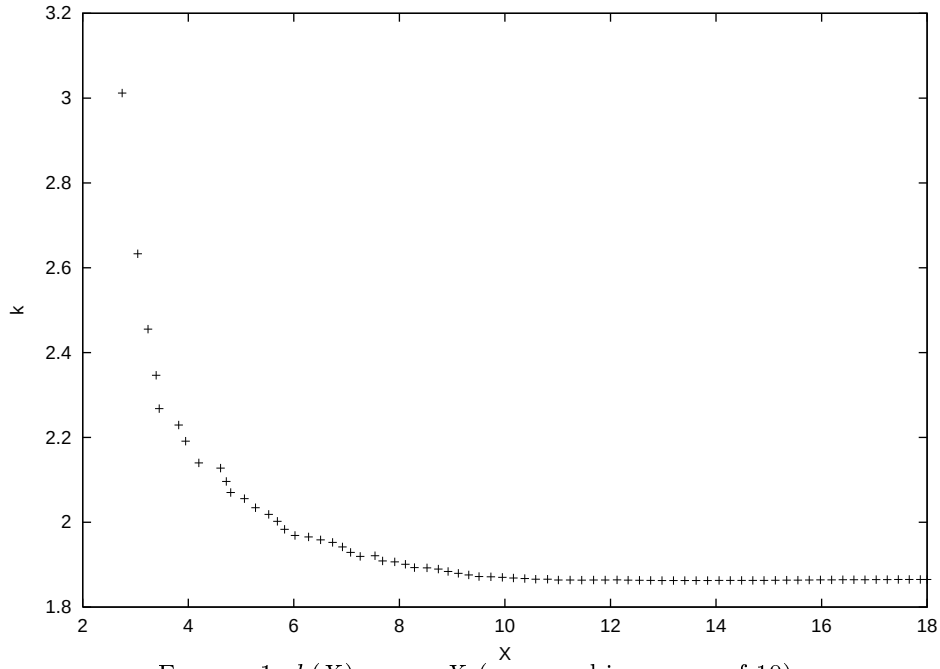

FIGURE 1. $k(X)$ versus X (expressed in powers of 10).

n	$k\,(10^n)$	$C(10^n)/C(10^{n-1})$
3	2.93319	
4	2.19547	7.000
5	2.07632	2.286
6	1.97946	2.688
7	1.93388	2.441
8	1.90495	2.429
9	1.87989	2.533
10	1.86870	2.396
11	1.86421	2.330
12	1.86377	2.286
13	1.86240	2.339
14	1.86293	2.319
15	1.86301	2.353
16	1.86406	2.335
17	1.86472	2.373
18	1.86522	2.394

TABLE 5. The function $k(X)$ and growth of $C(X)$ for $X = 10^n$, $n \leq 18$.

The Carmichael numbers up to 10^{17} with Lehmer index ≥ 2 are listed in Table 11.

n	$\log C\left(10^n\right)/(n\log 10)$
4	.21127
5	.24082
6	.27224
7	.28874
8	.30082
9	.31225
10	.31895
11	.32336
12	.32633
13	.32962
14	.33217
15	.33480
16	.33700
17	.33926
18	.34148

TABLE 6. $C(X)$ as a power of X.

m	c	25.10^9	10^{11}	10^{12}	10^{13}	10^{14}	10^{15}	10^{16}	10^{17}
5	0	203	312	627	1330	2773	5814	12200	25523
	1	1652	2785	6575	15755	37467	90167	215713	520990
	2	82	154	327	702	1484	3048	6094	12912
	3	102	172	344	725	1463	3059	6285	12845
	4	124	182	368	767	1519	3124	6391	13085
7	0	401	634	1334	2774	5891	12691	27550	60602
	1	1096	1885	4613	11447	28001	69131	168856	414999
	2	105	186	432	967	2109	4599	10011	21961
	3	152	232	496	1055	2178	4707	10039	21918
	4	129	211	450	985	2122	4592	9944	21832
	5	138	222	454	1033	2224	4777	10125	21911
	6	142	235	462	1018	2181	4715	10158	22132
11	0	335	547	1324	3006	7032	16563	38576	90731
	1	640	1131	2770	6786	16548	40891	100071	248840
	2	139	217	473	1068	2361	5338	12054	27268
	3	142	220	457	1045	2348	5319	12186	27483
	4	104	187	442	1026	2317	5261	11917	27276
	5	152	243	466	1066	2370	5316	12194	27670
	6	116	198	440	1061	2400	5384	12155	27362
	7	122	195	458	1023	2223	5165	11853	26960
	8	129	222	475	1107	2450	5449	12012	27639
	9	131	218	465	1042	2285	5179	11835	27158
	10	153	227	471	1049	2372	5347	11830	26968
12	1	2071	3462	7969	18761	43760	103428	243382	579215
	3	0	0	1	2	2	5	5	7
	5	20	32	64	124	228	448	805	1470
	7	47	75	147	289	547	1027	1906	3589
	9	25	36	60	103	165	294	560	1018
	11	0	0	0	0	4	10	25	56

TABLE 7. The number of Carmichael numbers in each class modulo m for $m = 5$, 7, 11 and 12.

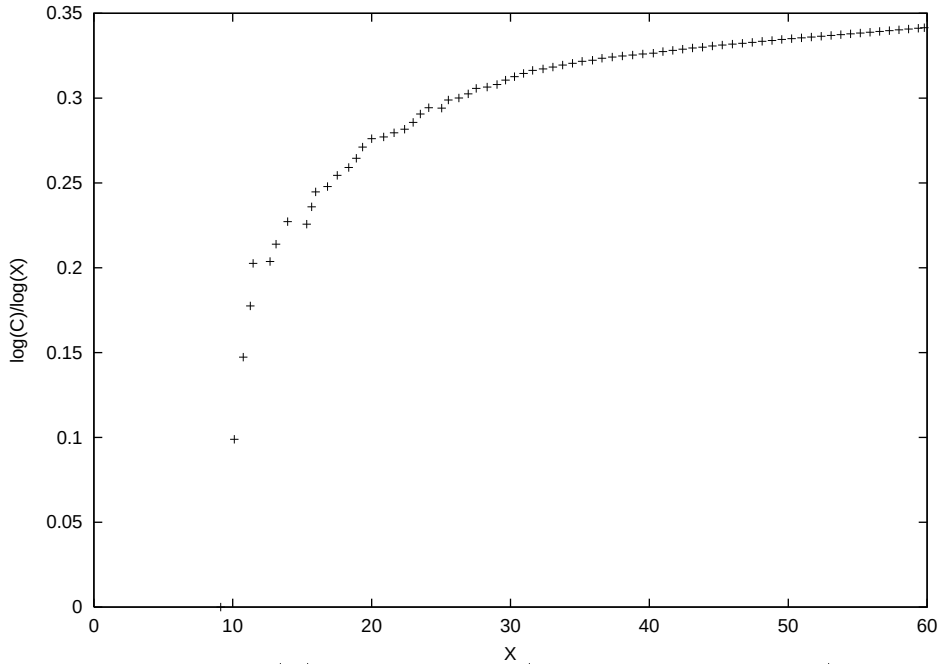

FIGURE 2. $C(X)$ as a power of X (expressed in powers of 10).

p	25.10^9	10^{11}	10^{12}	10^{13}	10^{14}	10^{15}	10^{16}	10^{17}	10^{18}
3	25	36	61	105	167	299	565	1025	1967
5	203	312	627	1330	2773	5814	12200	25523	53856
7	401	634	1334	2774	5891	12691	27550	60602	133380
11	335	547	1324	3006	7032	16563	38576	90731	214147
13	483	807	1784	3998	9045	20758	47785	110320	258539
17	293	489	1182	2817	6640	16019	38302	91783	221430
19	372	608	1355	3345	7797	18638	44389	106273	256020
23	113	207	507	1282	3135	7716	18867	46612	116538
29	194	336	832	2094	5158	12721	31110	76647	190486
31	335	571	1320	3086	7270	17382	41440	99845	242813
37	320	535	1270	2926	6826	16220	38647	92744	224582
41	227	390	1001	2418	5896	14344	34759	84977	209323
43	184	296	772	1920	4663	11594	28650	70904	176180
47	53	80	199	492	1223	2873	6810	17002	42440
53	92	160	351	813	2041	5143	12256	30560	76608
59	26	41	92	262	644	1611	3959	9613	23883
61	269	453	1075	2542	6047	14429	34503	83268	202835
67	110	178	407	1063	2540	6306	15295	37521	93475
71	104	194	521	1320	3351	8546	21485	53857	136174
73	198	348	849	2145	4925	11929	29072	71137	174025
79	64	107	247	686	1728	4318	10693	26766	66867
83	14	24	56	137	340	838	1929	4640	11465
89	68	131	320	788	1951	4981	12178	30737	77657
97	123	193	495	1277	3123	7594	18706	45542	112300

TABLE 8. Primes occurring in Carmichael numbers.

p	25.10^9	10^{11}	10^{12}	10^{13}	10^{14}	10^{15}	10^{16}	10^{17}	10^{18}
3	25	36	61	105	167	299	565	1025	1967
5	202	309	624	1325	2765	5797	12175	25481	53776
7	364	579	1218	2557	5461	11874	25915	57459	127453
11	263	428	1071	2509	5979	14397	33893	80745	192905
13	237	431	1058	2462	5699	13514	32025	76256	183523
17	117	206	496	1318	3244	8114	20206	50170	125098
19	152	244	532	1401	3358	8141	20020	49413	122452
23	37	78	207	535	1360	3317	8195	20803	53549
29	55	103	284	729	1822	4659	11577	29149	73645
31	101	168	390	876	2116	5153	12575	30667	75541
37	60	95	219	551	1401	3418	8594	21382	52989
41	35	68	171	414	1092	2736	6788	17275	43376
43	35	65	168	403	943	2308	5520	13636	34030
47	14	16	36	81	195	459	1135	2854	7171
53	19	30	55	147	363	973	2327	5842	14731
59	2	4	11	43	100	272	618	1542	3825
61	34	58	148	364	851	1978	4722	11278	27910
67	8	18	50	123	317	815	1950	4843	12109
71	15	25	66	161	389	979	2480	6178	15602
73	14	28	68	175	406	1015	2508	6277	15612
79	4	10	17	66	175	467	1163	2873	7002
83	1	1	4	8	39	79	175	457	1143
89	10	16	23	55	148	409	1003	2523	6293
97	10	20	50	106	261	606	1413	3445	8448

TABLE 9. Primes occurring as least prime factor in Carmichael numbers.

i	N	factors
5	6601	$7 \cdot 23 \cdot 41$
7	561	$3 \cdot 11 \cdot 17$
18	42018333841	$11 \cdot 47 \cdot 1049 \cdot 77477$
18	55462177	$17 \cdot 23 \cdot 83 \cdot 1709$
18	8885251441	$11 \cdot 47 \cdot 1109 \cdot 15497$
21	10585	$5 \cdot 29 \cdot 73$
22	2465	$5 \cdot 17 \cdot 29$
23	1105	$5 \cdot 13 \cdot 17$
25	11921001	$3 \cdot 29 \cdot 263 \cdot 521$
31	62745	$3 \cdot 5 \cdot 47 \cdot 89$
37	11972017	$43 \cdot 433 \cdot 643$
37	67902031	$43 \cdot 271 \cdot 5827$
39	334153	$19 \cdot 43 \cdot 409$
43	52633	$7 \cdot 73 \cdot 103$
44	15841	$7 \cdot 31 \cdot 73$
45	8911	$7 \cdot 19 \cdot 67$
47	2821	$7 \cdot 13 \cdot 31$
48	1729	$7 \cdot 13 \cdot 19$
49	1208361237478669	$53 \cdot 653 \cdot 26479 \cdot 1318579$
50	4199932801	$29 \cdot 499 \cdot 503 \cdot 577$
52	206955841	$17 \cdot 71 \cdot 277 \cdot 619$
53	1271325841	$17 \cdot 31 \cdot 179 \cdot 13477$
54	4169867689	$13 \cdot 29 \cdot 383 \cdot 28879$
55	271794601	$13 \cdot 19 \cdot 743 \cdot 1481$
60	6840001	$7 \cdot 17 \cdot 229 \cdot 251$
61	1962804565	$5 \cdot 103 \cdot 149 \cdot 25579$
67	11985924995083901	$29 \cdot 101 \cdot 1427 \cdot 16349 \cdot 175403$
67	410041	$41 \cdot 73 \cdot 137$
70	162401	$17 \cdot 41 \cdot 233$
76	4752717761	$11 \cdot 17 \cdot 107 \cdot 173 \cdot 1373$
81	35575075809505	$5 \cdot 197 \cdot 223 \cdot 353 \cdot 458807$
82	1496405933740345	$5 \cdot 47 \cdot 317 \cdot 40253 \cdot 499027$
83	142159958924185	$5 \cdot 37 \cdot 107 \cdot 58379 \cdot 123017$
83	158664761899885	$5 \cdot 37 \cdot 107 \cdot 53987 \cdot 148469$
83	204370370140285	$5 \cdot 37 \cdot 107 \cdot 48677 \cdot 212099$
83	24831908105124205	$5 \cdot 29 \cdot 719 \cdot 3023 \cdot 78790717$
85	171189355538562901	$23 \cdot 71 \cdot 983 \cdot 1031 \cdot 103437589$
90	3778118040573702001	$11 \cdot 47 \cdot 1051 \cdot 67967 \cdot 102302009$
92	520178982961	$11 \cdot 29 \cdot 131 \cdot 607 \cdot 20507$
94	12782849065	$5 \cdot 7 \cdot 269 \cdot 317 \cdot 4283$
95	8956911601	$11 \cdot 17 \cdot 127 \cdot 131 \cdot 2879$
97	5472940991761	$199 \cdot 241 \cdot 863 \cdot 132233$
97	721574219707441	$167 \cdot 241 \cdot 5039 \cdot 3557977$
97	83565865434172201	$103 \cdot 1993 \cdot 9551 \cdot 42622169$
97	9729822470631481	$127 \cdot 409 \cdot 110681 \cdot 1692407$
99	438253965870337	$43 \cdot 139 \cdot 49409 \cdot 1484009$

TABLE 10. Carmichael numbers with small index

ℓ	N	factors
2.00163	3852971941960065	$3\cdot5\cdot23\cdot89\cdot113\cdot1409\cdot788129$
2.00388	655510549443465	$3\cdot5\cdot23\cdot53\cdot389\cdot2663\cdot34607$
2.00438	978502408508476545	$3\cdot5\cdot23\cdot89\cdot173\cdot197\cdot13553\cdot68993$
2.00837	13462627333098945	$3\cdot5\cdot23\cdot53\cdot197\cdot8009\cdot466649$
2.01001	26708253318968145	$3\cdot5\cdot17\cdot113\cdot57839\cdot16025297$
2.01548	121719617715279585	$3\cdot5\cdot17\cdot89\cdot5897\cdot6833\cdot133103$
2.02272	371717470696746945	$3\cdot5\cdot29\cdot47\cdot113\cdot197\cdot353\cdot449\cdot5153$
2.03067	269040992399565	$3\cdot5\cdot23\cdot29\cdot4637\cdot5799149$
2.03207	158353658932305	$3\cdot5\cdot17\cdot89\cdot149\cdot563\cdot83177$
2.03614	1817671359979245	$3\cdot5\cdot23\cdot29\cdot359\cdot11027\cdot45893$
2.05825	184882434303977985	$3\cdot5\cdot17\cdot47\cdot113\cdot449\cdot304041473$
2.07138	16057190782234785	$3\cdot5\cdot17\cdot29\cdot269\cdot6089\cdot1325663$
2.07294	75131642415974145	$3\cdot5\cdot23\cdot29\cdot53\cdot617\cdot9857\cdot23297$
2.08045	202703674932642885	$3\cdot5\cdot23\cdot29\cdot53\cdot197\cdot3389\cdot572573$
2.08129	881715504450705	$3\cdot5\cdot17\cdot47\cdot89\cdot113\cdot503\cdot14543$
2.08283	462937246809774465	$3\cdot5\cdot17\cdot29\cdot107\cdot65729\cdot8901089$
2.08287	781506906921758865	$3\cdot5\cdot17\cdot23\cdot32117\cdot57077\cdot72689$
2.08309	224811969542371905	$3\cdot5\cdot17\cdot29\cdot113\cdot2297\cdot6977\cdot16787$
2.08381	31454143858820145	$3\cdot5\cdot17\cdot23\cdot2129\cdot39293\cdot64109$
2.08894	6128613921672705	$3\cdot5\cdot17\cdot23\cdot353\cdot7673\cdot385793$
2.09063	220629545178715905	$3\cdot5\cdot17\cdot23\cdot353\cdot1409\cdot7673\cdot9857$
2.09805	947087538769733505	$3\cdot5\cdot17\cdot23\cdot173\cdot929\cdot2237\cdot449153$
2.09841	12301576752408945	$3\cdot5\cdot23\cdot29\cdot53\cdot113\cdot197\cdot1042133$
2.11432	1886616373665	$3\cdot5\cdot17\cdot23\cdot83\cdot353\cdot10979$
2.11493	180950795673242145	$3\cdot5\cdot17\cdot23\cdot83\cdot353\cdot2663\cdot395429$
2.11493	3193231538989185	$3\cdot5\cdot17\cdot23\cdot113\cdot167\cdot2927\cdot9857$
2.12088	916541202603455265	$3\cdot5\cdot17\cdot23\cdot89\cdot347\cdot353\cdot947\cdot15137$
2.13936	11947816523586945	$3\cdot5\cdot17\cdot23\cdot89\cdot113\cdot233\cdot617\cdot1409$
2.14083	101817952350880305	$3\cdot5\cdot17\cdot23\cdot89\cdot113\cdot149\cdot3257\cdot3557$
2.23494	171800042106877185	$3\cdot5\cdot17\cdot23\cdot29\cdot53\cdot89\cdot197\cdot1086989$

TABLE 11. Carmichael numbers with Lehmer index greater than 2

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